Improved Generalized Inverse Beamforming for Jet Noise

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Outline

• Problem
• Mathematical formulation
• Low-TRL validation test results
  • Out-of-phase speakers
  • TUBE
  • Barrier
  • Isolated jet
  • Rectangular jet
  • Jet Blast Deflector
Problem
Mathematical Formulation
Nearfield Acoustic Holography

Williams

\[ \tilde{\psi}(x, y, z) = \mathcal{F}^{-1} \left( \tilde{\psi}(k_x, k_x, z_H) \right) \begin{cases} e^{ik_x(z-z_H)}, & k_x^2 + k_y^2 < k^2 \\ e^{-k_z(z-z_H)}, & k_x^2 + k_y^2 > k^2 \end{cases} \]
Generalized Inverse Beamforming

Suzuki

\[ \mathbf{v}_i = \mathbf{Aa}_i \]

\[ \mathbf{v}_i = \sqrt{\lambda_i} \mathbf{u}_i \]

Eigenvector component of CSM

Vector of source amplitudes for eigenvector \( i \)

Transfer matrix between sources and receivers

\[ \mathbf{a}_i \approx \mathbf{A}^\dagger \left( \mathbf{A} \mathbf{A}^\dagger + \varepsilon \mathbf{I} \right)^{-1} \mathbf{v}_i \quad \text{Underdetermined case} \]

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Array measured pressure (N-vector)
\[ p = GA \]

Far field points
\[ p_{\text{far field}} = HA \]

Alternate source model (M-vector)
\[ A = Ls \]

Jet source model (M-vector)
\[ L = \text{smoothing operator (regularization)} \]
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Cross spectral matrices

\[ C = \langle pp' \rangle \]
\[ C_A = \langle AA' \rangle \]

\[ C_{\text{far field}} = \langle HA (HA)' \rangle = H C_A H' \]

\[ C_s = \langle ss' \rangle \]
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Generalized inverse models

\[ C_{A_{LSSM}} = \langle A_{LSSM} A_{LSSM}' \rangle = \langle G^+ p (G^+ p)' \rangle = \langle G^+ pp' G^+ \rangle = G^+ \langle pp' \rangle G^+ = G^+ CG^+ \]

\[ G^+ = \text{Moore-Penrose generalized inverse of } G \]

computed using the singular value decomposition
Relationship of Beamforming and Generalized Inverse

\[ G = U \Sigma V' \]

\[ C_A^{\text{LSSMN}} = V \Sigma^{-1} U' C U \Sigma^{-1} V' \]

\[ C_A^{\text{beamforming}} = V \Sigma U' C U \Sigma V' \]
Regularized Solution

\[ C_{\text{smoothing}} = L(\mathbf{GL})^+ C(\mathbf{GL})^+ L' \]


GINV abstract submitted for the 2011 Aeroacoustics Conference
Generalized Inverse Button: GINV
Low-TRL validation test results
Out-of-phase speakers
Out-of-phase speakers

FDBF

CF 126.5 BW 89.4 m
0.279 m
66.7 dB

CF 252.0 BW 176.1 Hz
0.279 m
69.0 dB

CF 502.0 BW 350.0 Hz
0.279 m
72.6 dB

CF 1002.0 BW 704.3 Hz
0.279 m
81.8 dB

CF 1992.1 BW 1592.0 Hz
0.279 m
81.6 dB

GINV

CF 126.5 BW 89.4 m
0.279 m
48.3 dB

CF 252.0 BW 176.1 Hz
0.279 m
69.0 dB

CF 502.0 BW 350.0 Hz
0.279 m
72.6 dB

CF 1002.0 BW 704.3 Hz
0.279 m
81.8 dB

CF 1992.1 BW 1592.0 Hz
0.279 m
83.5 dB
Directivity Calculation

- Speakers array
- Synthetic microphone
- 30 m

Diagram showing a speakers array and a synthetic microphone at a distance of 30 m.
30 m directivity of out-of-phase speakers
Thematic Uniaxial Bladeless Environment
Speaker arranged for spinning modes
Speaker arranged for radial modes
GINV
30 m
Far field
Barrier

0.28 m height, speaker 0.2 m from edge
Isolated jet

3.5 mm
Mach 1.34
Re = 138,000
St = (Freq. Hz)/10^5
Isolated jet directivity
Rectangular jet

Parallel

Perpendicular
Polar directivity of rectangular jet

Parallel

Perpendicular
Azimuthal directivity of rectangular jet

Parallel

Perpendicular
Jet Blast Deflector
Jet Blast Deflector

- a) 125 Hz O.B.
- b) 250 Hz O.B.
- c) 500 Hz O.B.
- d) 1 kHz O.B.
- e) 2 kHz O.B.

Frequency, kHz

Theta, degrees
Remove JBD
Remove flight deck
Conclusions

• Generalized Inverse Method
  • Improved mathematical formulation
  • New regularization
  • Works better than expected
• Speakers
  • Possible to characterize in warehouse
• TUBE
  • Duct modes visible (first time?)
• Supersonic jet noise
  • See large scale structures properly (first time?)
• Rectangular jet
  • Asymmetrical directivity pattern
• JBD and flight deck
  • JBD scatters sound
  • Flight deck increases sound